

Effect of Coupled Heave-Pitch Motion on a Vessel Moving in Regular Water Ways

¹Harry-Ameh, E.; ²Orji, C.U and ³Dick, I.F

¹Rivers State University, Port Harcourt, Nigeria.

Submitted: 05-11-2021

Revised: 12-11-2021

Accepted: 15-11-2021

ABSTRACT: This work focusses on the effect of coupled heave-pitch motion on a vessel moving in regular water ways with the governing equation developed to describe this motion, considering the dynamic behavior of vessels navigating in harsh environmental conditions. Consequently the identification and description of hydrodynamic parameters in the equation of motion describing the coupled –heave-pitch motion was considered, and the Korvin-Kroukkovsky and Jacob’s strip theory was used in computation of excitation forces and moments for the coupling. A MATLAB program was developed to compute both coupled heave – pitch motion and uncoupled heave and pitch motion of the model specified and the sensitivity of heading angles was investigated on force, moment, heave, and pitch on the model unit. The results gotten from the MATLAB program for the coupled heave-pitch motion and the uncoupled heave and pitch motion of the model at head angle of 180° are 0.0372ft (0.0113m) and 0.0173rad for heave and pitch responses for the coupled condition while the values are much lower for the uncoupled condition with heave and pitch responses obtained as 0.0073ft (0.0022m) and 0.0164rad respectively. The MATLAB program was validated after been compared with the manual computations and results gotten for the model with 95% accuracy obtained, before the program was used to analyze the effect of coupled heave pitch motion of the Anchor Handling Towing Ship (WINPOSH REGENT) in regular water ways. This further gives credence to the fact that in motion analysis of a vessel in regular water ways, the effect of heave-pitch coupling can never be neglected.

KEYWORDS:Heave, Pitch, Coupling, Vessel, Regular waves, Hydrodynamic coefficients, Force, Moment, Sea keeping.

I. INTRODUCTION

Generally ships motion are defined by the six degrees of freedom that a ship, boat or any other

offshore vehicle can experience as it moves in water ways. The ships special axes are divided into three namely: the longitudinal, the transverse and the vertical axes. The movement of the vessel about these axes classifies the ships motion into two major groups namely the translational/displacement motion (heave, sway, and surge) and rotational /angular motion (yaw, pitch, and roll) as shown in Figure 1.

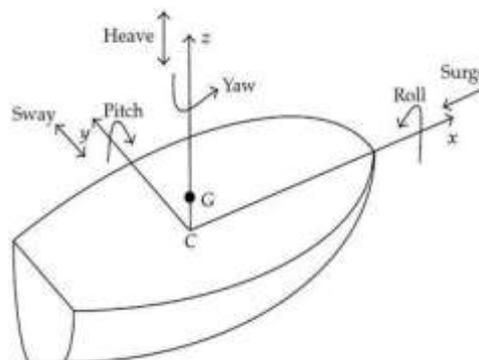


Figure 1. Ship Schematic Diagram showing the Six Degrees of Freedom.[1]

In order to gain sufficient knowledge or simplify the study of ships motion, some literatures consider only one degree of freedom as to have a ‘feeling’ for the forces and moments of the forces involved. But in actual seaways, the ship experiences all six degrees of freedom of motion.

However, the study of these ships coupled motion is very difficult as such, investigations are often restricted to some coupled motions such as:

(a) Heave and Pitch (b) Yaw and Sway (c) Yaw, Sway and Roll (d) Roll, Yaw and Pitch.[2].

The various motion of the vessel have different effects on the vessel which in turn has effects on humans, systems and mission capability. The effect of heave pitch coupling for a vessel

moving in regular water ways is important in sea keeping, as sufficient knowledge of this coupled motion, will be helpful in improving the design of the vessel for rough weather conditions. In this paper, a mathematical foundation for the derivation of the equations for coupled heave–pitch motion will be developed. Both numerical computations and MATLAB Program will be developed and used to carry out the rigorous mathematical procedures.

Problem Statement

The identification of the hydrodynamic parameters in the equations of motion describing the

II. MATERIALS AND METHODS

In analyzing the effect of heave and pitch coupling, the approach used in this paper was the Korvin-Kroukovsky and Jacob’s strip theory which major objective is to predict ships motion of pitch and heave for a seaway with given characteristics of amplitude and frequency. The Korvin-Kroukovsky and Jacob’s strip theory was the most suitable approach for this analysis as it helps in simplifying this complex motion into a mathematical model that can be easily handled following some basic computational procedures.

Solutions of the Coupled motion equation in complexform

Owing to the fact that the solutions to the motion equations comprises of both amplitudes and phase lags, they are best written in complex form. Therefore, let \bar{M} and \bar{F} represent the forcing functions in complex form:

$$\bar{F} = F_0 e^{i\sigma} \quad (1)$$

$$\bar{M} = M_0 e^{i\tau} \quad (2)$$

Let P,Q,R and S represent the complex forms of related coefficients from the equations of motion.

$$P = -(m + a_z)\omega^2 + iB\omega + c; \quad (3)$$

$$Q = -d\omega^2 + ie\omega + h \quad (4)$$

$$S = -(I_y + A_{yy})\omega^2 + iB\omega + c; \quad (5)$$

$$R = -D\omega^2 + iE\omega + H \quad (6)$$

Considering \bar{z} to represent all derivatives of z and $\bar{\theta}$ to represent all derivatives of θ , the equation of

coupled heave–pitch motions of a ship in a realistic seaway can provide a method for the accurate estimate of ship response in realistic seas. This will also provide a tool for validating the results obtained from programs based on the strip theory or three-dimensional potential flow[3].

Aim

The aim of this paper is to develop a predicting procedure / tool to estimate the effect of heave pitch coupling which can provide a useful tool aboard ships to help ship’s for navigational decision making.

motion in the general operator form can be written as:

$$\text{First equation (for heave): } P\bar{z} + Q\bar{\theta} = \bar{F} \quad (7)$$

$$\text{Second equation (for pitch): } S\bar{\theta} + R\bar{z} = \bar{M} \quad (8)$$

Where \bar{z} and $\bar{\theta}$ represents the complex amplitudes of motion

$$\bar{z} = z_a e^{i\delta} \quad (9)$$

$$\bar{\theta} = \theta_a e^{i\epsilon} \quad (10)$$

We obtain \bar{z} and $\bar{\theta}$:

$$\bar{z} = \frac{\bar{F}-Q\bar{\theta}}{P} \quad (11)$$

$$\bar{\theta} = \frac{\bar{F}-P\bar{z}}{Q} \quad (12)$$

Also considering the pitch equation:

$$\bar{z} = \frac{\bar{M}-S\bar{\theta}}{R} \quad (13)$$

$$\bar{\theta} = \frac{\bar{M}-R\bar{z}}{S} \quad (14)$$

Equating \bar{z} and $\bar{\theta}$ from their respective equations yields

$$\frac{\bar{F}-Q\bar{\theta}}{P} = \frac{\bar{M}-S\bar{\theta}}{R} \quad (15)$$

$$\frac{\bar{F}-P\bar{z}}{Q} = \frac{\bar{M}-R\bar{z}}{S} \quad (16)$$

Solving for \bar{z} and $\bar{\theta}$ in the above equations, yields the solution for complex pitch and heave:

$$\bar{z} = \frac{\bar{M}Q-\bar{F}S}{QR-PS} \quad (17)$$

$$\bar{\theta} = \frac{\bar{F}R-\bar{M}P}{QR-PS} \quad (18)$$

Having found amplitudes and phase lags for \bar{z} and $\bar{\theta}$, the final solutions for the two motion equations can be expressed as:

$$\bar{z} = z_a e^{i\delta} = z_a (\cos \delta + i \sin \delta) \quad (19)$$

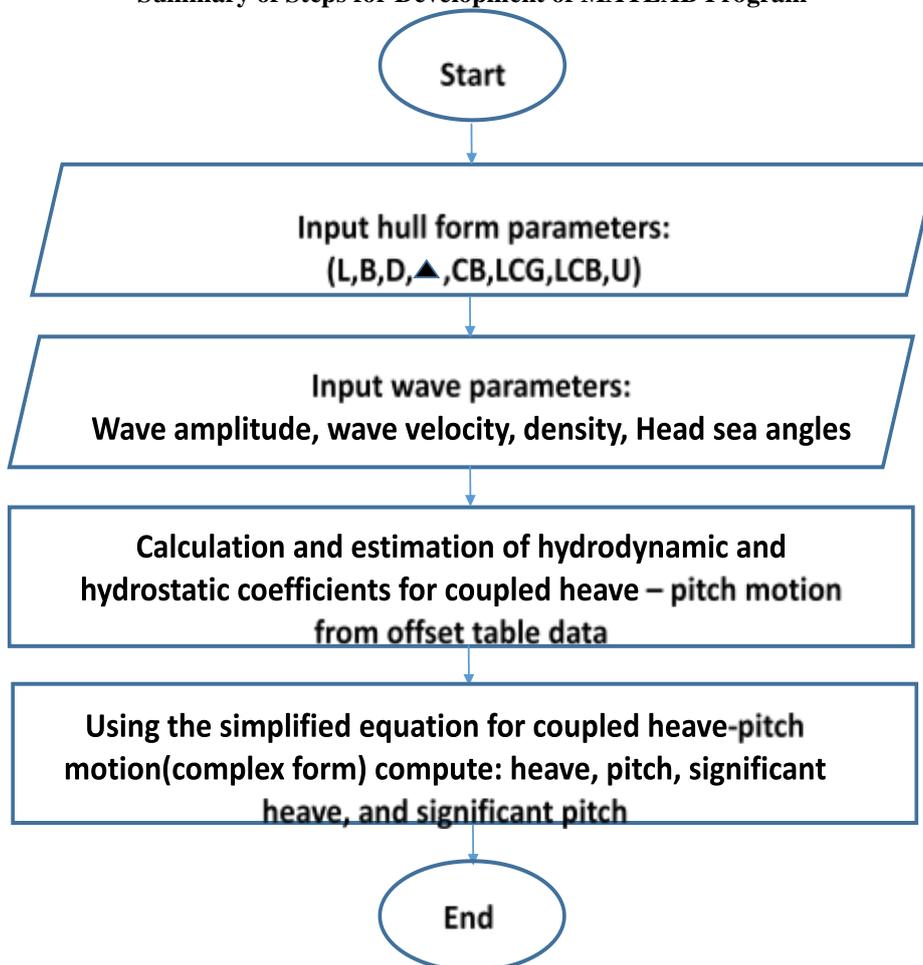
$$\bar{\theta} = \theta_a e^{i\varepsilon} = \theta_a (\cos \varepsilon + i \sin \varepsilon) \quad (20)$$

Where z_a and δ heave amplitude and phase lag; θ_a and ε are pitch amplitude and phase lag.

It appears that the final solution from Equations (19) and (20) known as the Korvin-Kroukovsky's equation, is a reasonable solution that can be used to predict the complex motion of pitch and heave owing to the numerous simplifications made to arrive at the mathematical model that can be easily handled. This equation can be applied in terms of a given seaway of the form:

$$\zeta = \zeta_a \sin k(x - V_w t) \text{ or } \zeta_a \sin \omega_e t.$$

Summary of Steps for Development of MATLAB Program



III. RESULTS AND DISCUSSION

The model presented in the Table 4.1 with its' hydrodynamic parameters will serve as the bench mark or basis for validating the Effect of coupled Heave-Pitch motion of the Anchor Handling Towing Ship (AHTS) WINPOSH REGENT with vessel parameters and hydrodynamic

characteristics stated in Table 1. Whilst this is a typical sample of the computational procedure, the results pertain only to the given vessel, speed, and wavelength. This is worthy of noting especially when phase angles of wave, heaving force and pitching moment are under considerations.

Table 1 Model Parameters and Hydrodynamic Characteristics

Parameter	Symbol	Value
Length of model	L_w	19.2ft (5.85m)
Maximum beam	B_m	2.592ft (0.79m)
Draft	T	1.144ft (0.349m)
Displacement	Δ	2837.76lb (1287.18kg)
Block coefficient	C_B	0.80
LCG	LCG	+0.48ft (0.146m) (forward of mid-ship)
LCB	LCB	+0.48ft (0.146m) (2.5% L_{pp} forward of mid-ship)
Model speed	U	4.788ft/sec (1.459m/s)
Radius of gyration	k_{yy}	4.588ft (1.4m)
Wave amplitude	ζ_a	0.2ft (0.06m)
Wave velocity	V_w	9.90ft/sec (3.02m/s)
Density	ρ	1.9384lb-sec ² /ft ⁴ (fresh water at 59°F) (999.71kg/m ³)
Angle at head sea	μ	180°

Parameter	Symbol	Value
Length of model	L_w	61.20m
Maximum beam	B_m	16.60m
Draft	T	4.50m
Displacement	Δ	2591tons (2591000kg)
Block coefficient	C_B	0.80
LCG	LCG	+0.146m (forward of mid-ship)
LCB	LCB	+0.146m (2.5% L_{pp} forward of mid-ship)
Model speed	U	14knots (1.459m/s)
Radius of gyration	k_{yy}	1.4m
Wave amplitude	ζ_a	0.06m
Wave velocity	V_w	3.02m/s
Density	ρ	(fresh water at 59°F) (999.71kg/m ³)
Angle at head sea	μ	180°

Table 2: Vessel Parameter and Hydrodynamic Characteristics for AHTS WINPOSH REGENT

Parameters	Manual calculation of model parameters at 180° head angle	MATLAB result of model parameter at 180° head angle	Percentage Error (%)
Force	9.9061	9.6510	2.575181
Moment	438.81	437.3734	0.327385
Heave	0.039	0.0373	4.358974
Pitch	0.018	0.0173	3.888889

Table 3: Validation of Computational Procedure coded in MATLAB

From the Table 3 above it is seen that for all the parameters specified, none of the percentage errors exceeds 4%. This shows that the MATLAB program generated has about 95% accuracy and as such can be used to analyze and validate the effect of coupled heave-pitch motion on the vessel specified.

Head angles (Degrees)	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
FO (N)	17635	16396	14068	12570	12052	11296	8964.9	4662	5246.9	13539	22089	28365	30669
M0 (N-m)	5926	5385	4279	3594	3041	3040	3073.9	3065	3281.7	4006	5043	5919	6256
PITCH (rad)	0.0030	0.0012	0.0001	0.0025	0.0044	0.0051	0.0034	0.0014	0.0013	0.0028	0.0041	0.0049	0.0052
HEAVE (m)	0.1069	0.0612	0.0220	0.0099	0.0057	0.0031	0.0016	0.0011	0.0011	0.0017	0.0026	0.0032	0.0034
SIG. HEAVE	0.3038	0.1772	0.0633	0.0286	0.0167	0.0087	0.0046	0.0032	0.0031	0.0050	0.0072	0.0092	0.0100
SIG. PITCH	0.0085	0.0033	0.0027	0.0072	0.0129	0.0148	0.0098	0.0040	0.0038	0.0079	0.0118	0.0143	0.0148

Table 4 : Results for Coupled Heave-Pitch Motion Data with varying Head angles (Vessel)

Head angles (Degrees)	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
FO (N)	0.5292	0.5796	0.7274	0.9625	1.2690	1.6258	2.0088	2.3917	2.7486	3.0550	3.2901	3.4379	3.4884
M0 (N-m)	24.0079	26.2952	33.0012	43.6689	57.5713	73.7610	91.1346	108.5083	124.6980	138.6004	149.2681	155.9741	158.2614
PITCH (rad)	0.0000279	0.00003303	0.0000487	0.0000730	0.0001010	0.000137	0.0002089	0.0004	0.0006	0.0002400	0.0001561	0.0001262	0.0001182
HEAVE (m)	0.0000011	0.00000133	0.000002	0.0000029	0.0000040	0.000005	0.0000084	0.0000	0.0000	0.0000084	0.0000056	0.0000045	0.0000042
SIG. HEAVE	0.0000033	0.00000384	0.0000057	0.0000083	0.0000114	0.000015	0.0000241	0.0001	0.0001	0.0000244	0.0000158	0.0000131	0.0000121
SIG. PITCH	0.0000794	0.00009445	0.00014	0.0002064	0.0002932	0.000398	0.0006083	0.0013	0.0016	0.0006834	0.0004535	0.0003636	0.0003415

Table 5 : Results for Uncoupled Heave-Pitch Motion Data with varying Head angles (Vessel)

Result Discussion of Plot of Heave force against head angles:

From the results obtained, it is seen that the heave force amplitude is smallest at 180° and highest at 0° . This simply suggests that the heave force decreases as the angle of attack increases.

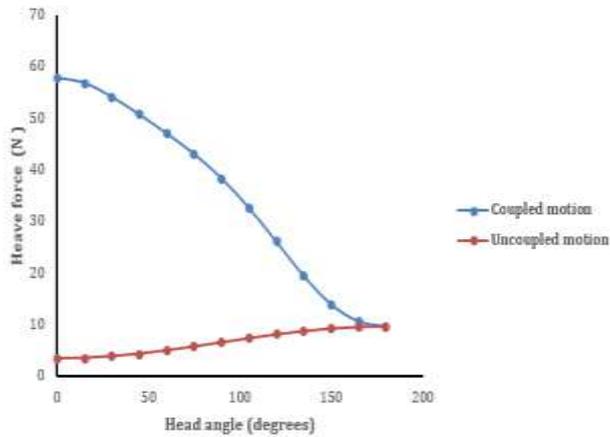


Figure 2 Heave force vs. Head angle

The figure above compares the heave force for the coupled motion and the uncoupled motion and it is clearly seen that the coupled motion values are much higher than the uncoupled motion at the respective head angles.

Result discussion on plot of pitch moment against head angle:

Pitch moment value is lowest at 180° and maximum at 0° , this suggests that the pitch moment decreases as the angle of attack increases.

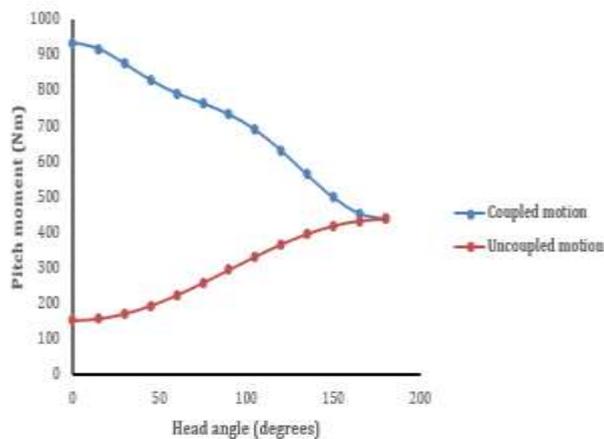


Figure 3 Pitch moment vs. Head angle

The figure above compares the pitch moment for the coupled motion and the uncoupled motion and it is clearly seen that the coupled motion values are much higher than the uncoupled motion at the respective head angles.

Plot of heave motion against angle of attack.

It was observed that heave motion was constant at some point (0° - 30°), gradually increased and got to maximum point at (105°) and dropped sharply down to a minimum value at 180° .

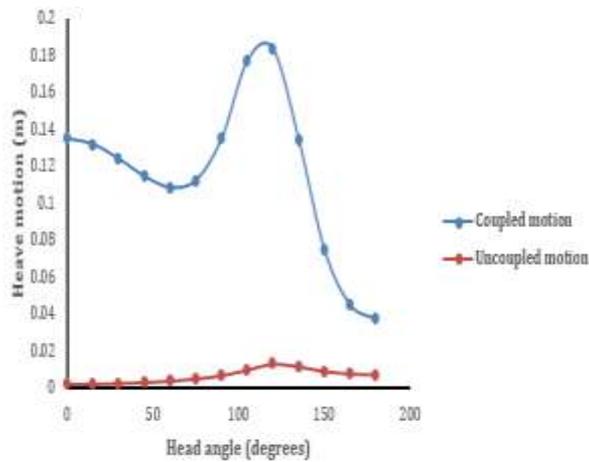


Figure 4: Heave motion vs. Head angle

The figure above compares the Heave for the coupled motion and the uncoupled motion and it is clearly seen that the coupled motion values are much higher than the uncoupled motion at the respective head angles.

Result discussion on plot of pitch motion against angle of attack

Pitch motion dropped initially as the angle of attack increased at some point (0° - 45°), then increased to maximum point at (105° at the mid-point), then dropped to its lowest value at 180° .

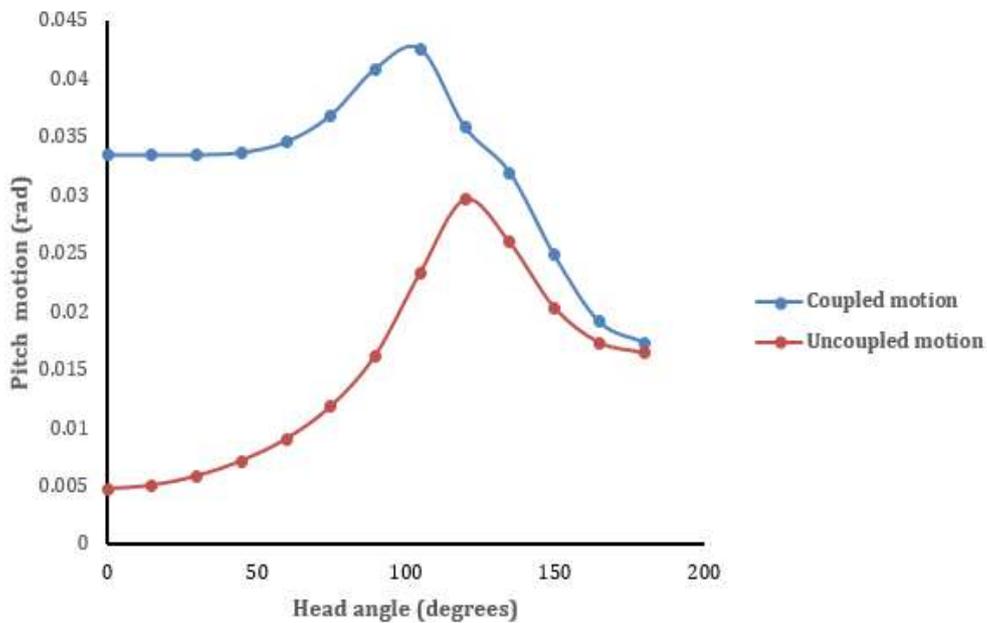


Figure 5 Pitch motion vs. Head angle

The figure above compares the pitch for the coupled motion and the uncoupled motion and it is clearly seen that the coupled motion values are much higher than the uncoupled motion at the respective head angles.

IV. CONCLUSION

From the result of the analysis stated especially comparing Table 4 and 5, it can be clearly seen that the values for force, moment, heave, pitch, significant heave and significant pitch are much higher for the coupled-heave pitch motion analysis than that of the uncoupled heave – pitch motion analysis. This further gives credence to the fact that in motion analysis of a vessel moving in regular water ways, the effect of heave-pitch coupling should be neglected and this is a major contribution to knowledge.

The work on dynamics of marine vehicles validates this thesis as the results obtained from the

MATLAB program developed for the model is quite satisfactory with the model results presented in the work of Bhattacharyya (1978).

SOME OF THE ADVANAGES FROM THE ABOVE RESULTS

- i. Development of two MATLAB programs to compute coupled heave-pitch motion and uncoupled heave and Pitch motion for ship motion analysis in regular waves.
- ii. From the programs developed it was further observed that the effect of heave-pitch coupling in ship motion analysis should not be neglected.

MATLAB SCRIPT

```
% enter ship principal parameter
p1=5; % number of sections used
step=32; % step size within one cycle for we*t estimate must be even number
SM=zeros(1,p1); % Simpson multiplier for num integration
L=61.2; % length of vessel
%B=16.60; % Beam of vessel
T=4.50; % Draft of vessel
g=9.81;
Delta=2591000;% Mass Displacement
CB=0.80; % Block Coefficient
LCG=0.146; % Longitudinal center of gravity
u=7.2; %ship speed
Kyy=1.4; %Radius of gyration
zeta=0.06; % wave amplitude
v=3.02; % wave speed
rho=999.71; % Density of fresh water at 59 degree Fahrenheit
%mu=[0:15:180]; %heading angles
mu=180;

%Enter station and accompanying parameters
St=[0,5,10,15,20];
Bmax=[0,16.6,16.6,16.6,0]; % Beam at different stations
Tn=[4.50,4.50,4.50,4.50,4.50]; % Draft at different stations
Sa=[0,74.7,74.7,74.64,0]; % Sectional Area at different stations
Wtpm=Bmax.*Tn.*rho.*g; % Weight per meter
lever=[-2.78,1.32,-0.15,-1.61,-3.07]; % Lever arm at different stations

% calculate wave frequency w
w=sqrt((2*pi*g)/L);
% enter wave frequency
% w=3,2;
%frequency of encounter we
we=w-(w^2*(u)*cosd(mu)/g);

% calculation for added heave mass
a7=Bmax.^2;
a1=(we^2)/(2*g)*Bmax;
a2=Bmax./Tn;
a3=Bmax.*Tn;
```

```
a4=Sa./a3;

for i=1:5
if isnan(a4(i))
a4(i)=0;
else
end
end
```

Enter value of Added mass coefficient a5 from appendix chart using a1,a2 and a4 estimated from routine 1 above

```
a5=[0,0.98,0.98,0.84,0];
a8=(rho*pi/8)*a7;
a2d33=a8.*a5;

% This section generates simpsons multiplier for the various stations
SM(1)=1; SM(p1)=1; % Allocate the two extreme values of 1, first and last values for Portside and Starboard
cordinates
for m1=2: (p1-1);if mod(m1,2)==0;SM(m1)=4; else
    SM(m1)=2;end
end
a9=a2d33.*SM;
a10=lever.*lever;
a11=a2d33.*a10;
a12=a11.*SM;
sumA=sum(a9);
sumB=sum(a12);
%sumA=a13;
%sumB=855.6036;
%Computing the Added Mass for Heaving (a33) and Added mass moment of inertia for Pitching
%(a55)
a33=1/3*we*sumA;
a55=1/3*we*sumB;

%%Enter parameters to obtain damping coefficients for heaving and pitching
% enter value of amplitude ratio for two-dimensional body in heaving b1 from appendix chart using a1,a2
% and a4 estimated from routine 1 above

b1=[0,0.57,0.57,0.66,0];
b2=b1.*b1;
b3=(rho*(g^2)/(we^3)).*b2;
b4=b3.*SM;
b5=b3.*a10;
b6=b5.*SM;
%b7=sum(b4);
%b8=sum(b6);
sum1=sum(b4);
sum2=sum(b6);

%Computing damping coefficient for Heaving (b33) and dampinp coefficient for Pitching
%(b55)
b33=1/3*we*sum1;
b55=1/3*we*sum2;
```

```

%%Enter parameters to obtain restoring force coefficients for heaving(c33)
%%and pitching(c55);
c1=rho*g.*Bmax;
c2=c1.*SM;
c3=c1.*a10;
c4=c3.*SM;
%c5=cumsum(c2);
%c6=cumsum(c4);
SUM1=sum(c2);% 1617.8;
SUM2=sum(c4);% 30193;
c33=1/3*we*SUM1; % restoring force coefficient for heaving
c55=1/3*we*SUM2;% restoring force coefficient for pitching

```

```

%%Enter parameters to obtain coupled term coefficients (d,e,h,D,E,H)
a23=a2d33.*lever;
a24=a23.*SM;
b27=b3.*lever;
b28=b27.*SM;
c15=c1.*lever;
c16=c15.*SM;
sum3=sum(a24);
sum4=sum(b28);
sum5=sum(c16);
% sum3=-8.9366;
% sum4=-70.2392;
% sum5=-776.6;
d=-1/3*we*sum3;
D=d;
e=-1/3*we*sum4+u*a33;
E=-1/3*we*sum4-u*a33;
h=-1/3*we*sum5+u*b33;
H=-1/3*we*sum5;

```

```

%%Calculating the mass (m) and Moment of Inertia (Iyy) of the vessel
mn=Wtpm./g;
m1=mn.*SM;
m2=mn.*a10;
m3=m2.*SM;
summ1=sum(m1);
summ2=sum(m3);
% summ1=55.4988;
% summ2=1112.6;
m=1/3*we*summ1;
Iyy=1/3*we*summ2;

```

```

%%Enter parameters to obtain exciting Forces and Moments (F and M)

```

```

k=2*pi/L;
k1=k.*lever;
k2=sin(k1);
k3=cos(k1);
Tm=Sa./Bmax;
for i=1:5
if isnan(Tm(i))
Tm(i)=0;

```

```
else
end
end
k4=k*Tm;
k41=exp(-k4);
k5=c1.*zeta;
k6=a2d33.*(-zeta)*(we^2);
k7=k5+k6;
% slope of added mass k8 obtained from sample calculation in routine
k8=[-1.043,-0.522,0.075,0.522,0.894];
k9=u*zeta*we.*k8;
k10=b3.*zeta*we;
k11=k10-k9;
k12=k7.*k2;
k13=k11.*k3;
k14=k12+k13;
k15=k7.*k3;
k16=k11.*k2;
k17=k15-k16;
k18=k14.*k41;
k19=k18.*SM;
k20=k17.*k41;
k21=k20.*SM;
k22=k18.*lever;
k23=k22.*SM;
k24=k20.*lever;
k25=k24.*SM;
sum6=sum(k19);
sum7=sum(k21);
sum8=sum(k23);
sum9=sum(k25);
%sum6=-4.1255;
%sum7=4.3633;
%sum8=231.0478;
%sum9=-144.3447;

%Calculating the exciting force component F1
F1=1/3*we*sum6;

%Calculating the exciting force component F2
F2=1/3*we*sum7;
%Calculating the amplitude of exciting force FO
FO=sqrt((F1^2)+(F2^2));
% Calculating phase angle of the exciting force for heaving An(sigma)
A=atand(F2/F1);
An=mu+A;% Phase angle of the exciting force acting on the vessel
wet=[0:2*pi/step:2*pi];
F=FO*cos(wet+An);

%Calculating the exciting Moment component M1
M1=1/3*we*sum8;
%Calculating the exciting moment component M2
M2=1/3*we*sum9;
%Calculating the amplitude of exciting Moment MO
MO=sqrt((M1^2)+(M2^2));
```

```
% Calculating phase angle of the exciting moment for pitching y(tau)
y=atand(M2/M1);
M=MO*cos(wet+y);
```

Motion Analysis Coupled Case(Complex Form)

```
Ft=complex(F1,F2);%Ft= -6.6193+7.0008i;
Mt=complex(M1,M2);% 370.7103-231.5974i;

Pr=-(m+a33)*we^2+c33;
Pi=(b33*we);
P=complex(Pr,Pi);
Sr=-(Iyy+a55)*we^2+c55;
Si=b55*we;
S= complex(Sr,Si);
Qr=-d*we^2+h;
Qi= e*we;
Q=complex(Qr,Qi);
Rr=-D*(we^2)+H;
Ri=E*we;
R=complex(Rr,Ri);
%calculating the heaving amplitude Za and Heave phase angle (o)
Zac=((Mt*Q)-(Ft*S))/((Q*R)-(P*S));
Zar=real(Zac);
Zai=imag(Zac);
Za=sqrt((Zar)^2+(Zai)^2);
sigmaH=atand(Zai/Zar);
%calculating the Pitching amplitude Ya(teta) and pitch phase angle epsilom(rn)
Yac=((Ft*R)-(Mt*P))/((Q*R)-(P*S));
Yacr=real(Yac);
Yaci= imag(Yac);
Ya=sqrt((Yacr)^2+(Yaci)^2);
r=atand(Yaci/Yacr);
rn=mu+r
%Wave amplitude (WaveA)
WaveA=zeta*sin(wet)

%Heaving motion
Z=Za*cos(wet+sigmaH)
%Pitch motion (Y)
Y=Ya*cos(wet+rn)

Zsigval=4*std(Z);
Ysigval=4*std(Y);

Data=[FO;MO;Ya;Za;Zsigval;Ysigval];
figure(1)
plot(wet,F); ylabel('Heave Force');xlabel('wet');
figure(2)9,10 (
plot(wet,M); ylabel('Pitch Moment');xlabel('wet');
figure(3)
plot(wet,Z); ylabel('Heave');xlabel('wet');
figure(4)
plot(wet,Y);ylabel('Pitch');xlabel('wet');
```

```
figure(5)
plot(wet,WaveA); ylabel('Wave amplitude');xlabel('wet');
% figure(6)
%plot(MU,F0RCE);ylabel('Heave Force');xlabel('mu');
% figure(7)
%plot(MU,MOMENT);ylabel('Pitch Moment');xlabel('mu');
% figure(8)
%plot(MU,HEAVE);ylabel('Heave Motion');xlabel('mu');
% figure(9)
%plot(MU,PITCH);ylabel('Pitch Motion');xlabel('mu');
```

REFERENCES

- [1]. Ibrahim, R.A, & Grace, I.M. (2010). Modelling of ship roll dynamics and its coupling with heave and pitch. Mathematical Problems in Engineering 2010
- [2]. Haddarra, M.R., & Cao, S. (1996). A study of the dynamic response of submerged rectangular flat plates. Marine Structures, 9 (10), 913-933
- [3]. Bhattacharyya R.(1978). Dynamics of Marine Vehicles. John Wiley & Sons Incorporated